A Revisit of SO(6) Dynamical Symmetry in Nuclear Structure

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Abstract

According to the analysis based on the fermion dynamical symmetry model, nuclei previously regarded as SO(6)-like (e.g. ^{128}Xe and ^{196}Pt) are shown to be more akin to the transitional nuclei between SO(7) and SO(6) symmetries.

The possible empirical evidence for the SO(6) dynamical symmetry, first in ^{196}Pt [1] and later in the Xe-Ba region [2] was one of the highlights in the early days of the interacting boson model (IBM). Some concerns were raised in the mid-eighties about this. The first was an insightful observation by Leviatan et al. [3] who stressed that the strikingly good agreement between the predicted and the measured B(E2) branching ratios can essentially be viewed as manifestation of the SO(5) selection rules. Since SO(5) is a common subgroup of SO(6) and U(5), most of these branching ratios cannot differentiate between the two parent symmetries [3]. The second is a note by Fewell [4] who pointed out that according to his analysis, which concentrated primarily on studying the absolute B(E2)'s and their branching ratios, the spectroscopy of ^{196}Pt (and presumably that means also Xe-Ba as well) is more akin to U(5) (i.e. vibrational) than SO(6) (γ-soft). In a subsequent paper, Casten and Cizewski (CC) contended that while they agree with the precautions of Ref [3], they disagree with Fewell's [4]. Specifically, CC stated that using the prescription of Fewell's, namely changing the sign of the $\tau(\tau + 3)$ term of the energy eigenvalue, the level scheme beyond 1.5 MeV does favor an SO(6) picture. More important, their arguments centered on the analysis of the B(E2)'s and the adherence of the branching ratios to the selection rule $\Delta\sigma = 0$ selection rule for the two highly excited 2^+ states (1604 (keV) and 1847 (keV)) for ^{196}Pt and concluded that this nucleus should "still" be SO(6)-like [5].

There are some assumptions which are implicit in CC's paper. The two aforementioned highly excited 2^+ states are assumed to be pure $\sigma = N - 2$ states of the SO(6) symmetry. Effects such as broken pair and mixed-symmetry are either assumed to be unimportant and hence can be ignored or somehow they play no role in the transitions. We are of course unaware of any convincing physical arguments which could alleviate the importance of these assumptions. There is an additional caveat worth noticing in the study of branching ratios for these highly excited states. Quite recently, Borner et al. [6] have measured the upper limit of the absolute $B(E2; 2_1^+ \rightarrow 0_1^+)$ in ^{196}Pt and found that there is an order of magnitude hindrance from the allowed SO(6) transitions. While the above discussed assumptions are not relevant for 0_3^+, it does point to the fact that these transitions are basically very weak

ones. Since the two excited 2^+ states lie even higher in energies, it is not expected that their absolute B(E2)'s for $\Delta\sigma = 2$ can be larger. Therefore, the weak absolute values of the B(E2) will undoubtedly dampen the significance of their ratios. Hence we feel that measuring the absolute B(E2)'s for these states should be crucial in this study. At this moment the situation is as follows: The CC analysis is certainly correct within the context of the IBM-1 but perhaps is incomplete and the window is opened for further analysis of the characteristics of the spectroscopes of ^{196}Pt and other so-called SO(6)-like nuclei.

The empirical manifestation of the boson dynamical symmetry is a strong incentive to examine whether at the fermionic level the symmetry can show up as well. For the present case, it is particularly intriguing because one may ask whether more can be learned from the fermion picture, since it appears from the CC analysis [5] that the IBM-1 point of view has been exhausted. The answer turns out to be positive and this paper intends to explore this and hence this revisit.

Our starting point of the analysis is the fermion dynamical symmetry model (FDSM) [7,8]. The details of the model was extensively discussed elsewhere. Suffice to mention that the IBM was very much its inspiration and the Ginocchio schematic fermion pairing-plus-quadrupole model [9] its structural underpinning. In a recent paper, we have argued that for the ^{196}Pt, although the symmetry of $Sp_\nu(6) \times SO_\pi(8)$ cannot mathematical accommodate an exact SO(6) symmetry, the results, by introducing the proper neutron-proton couplings and pairing forces, do suggest a remarkably accurate effective one [10]. So, although technically one should not use a pure SO(8) symmetry, which contains an exact SO(6) subgroup to analyze the data, we shall do so for our present purpose because of [10]. Of course, just as the CC analysis, our analysis was also incomplete because the assumptions we mentioned earlier were operative there as well.

We begin by first demonstrating that the data does not support the vibrational scheme. This should reinforce the CC conclusion. For the FDSM, the pure vibrational symmetry is SO(5) $\times SU(2)$. The $SU(2)$ here is the phonon symmetry. The eigenvalues of this dynamical symmetry (subtracting the SO(3) term) are $G_0\kappa(\Omega - \kappa + 1) + \alpha\tau(\tau + 3)$, where κ and τ

are the phonon and $SO(5)$ quantum numbers. In order for the system to have a phonon structure, the first term must dominate. In Fig.1 of ref. [11], the low lying levels (with the $SO(3)$ term subtracted) of ^{128}Xe and ^{132}Ba are in excellent agreement with $\tau(\tau+3)$, i.e. the $SO(5)$ scheme. This implies that the levels are non-vibrational.

Recently [11], we showed that by adding the pairing interaction to the pure $SO(6)$ Hamiltonian, the spectroscopes of ^{128}Xe and ^{196}Pt, which are considered to be the archetypical examples of $SO(6)$, can be better understood both within the FDSM and the IBM. That analysis was motivated by the so-called τ-compression effect [11]. Specifically, it means that for the high $SO(5)$ multiplets, the data show a significant compressed $\tau(\tau+3)$ behavior. By adding the pairing force, such a compression, in the language of the Coriolis antipairing effect, can be very naturally accommodated. However, for the FDSM, there is an additional interest here. It is well known that the $SO(6)$-plus-pairing description can preserve the E2 selection rules of the $SO(5)$ symmetry ($\Delta\tau =1$ for the strong transitions and $\Delta\tau=0, 2$ for the weak transitions) since the pairing interaction is an $SO(5)$ scalar and $SO(5)$ is a common subgroup for all three subgroups of $SO(8)$: $SO(6)$, $SO(5) \times SU(2)$ and $SO(7)$. (The $SO(7)$ of course has no straightforward counter part in the boson picture [12]). Since the $SO(5)$ is not the "sole property" of the $SO(6)$, one can only conclude that these E2 branching ratios are not inconsistent with an $SO(6)$ picture, but they are not necessarily exclusively due to it. Furthermore, it is known that in the $SO(8)$ symmetry, the quadrupole-quadrupole (QQ) interaction reflects the $SO(6)$ and the pairing the vibrational $SO(5) \times SU(2)$. Yet, unlike the IBM's, which has no natural dynamical symmetry between the γ-soft $-$ vibration leg of the Casten triangle [13], when the QQ interaction and pairing is exactly balanced, an $SO(7)$ emerges as shown in Fig.1. Hence, the $SO(6)$-plus-pairing description in [11] implies the presence of an $SO(7)$ symmetry or, at the very least, a transition between the $SO(7)$ and $SO(6)$.

In Fig.2, we present the typical $SO(6)$ and $SO(7)$ spectroscopes. The most striking difference for these two limits are the E2 transitions between those excited 0^+ and 2^+ states. We see that the $SO(6)$ predicts a very strong $0_2^+ \rightarrow 2_2^+$ transition ($\Delta\tau=1$ selection rule). It

should be mentioned that if empirically the transitions (i.e. $\Delta\tau = \pm 2$) such as $0_2^+ \to 2_1^+$ and $0_3^+ \to 2_2^+$ are weak but not zero, then they can be accommodated by adding the $(D^\dagger D)^2$ term in the E2 operator. Such a modification of the E2 operator was first introduced by Yoshida et al. [14] and is allowed within the FDSM since the use of the SO(6) generator as E2 operator is strictly one of convenience. Note that the inclusion of the $(D^\dagger D)^2$ will not drive the system towards SU(3) since this symmetry is not present in the SO(8).

The SO(7) predicts that both $0_2^+ \to 2_1^+$ and $0_3^+ \to 2_2^+$ are strong transitions. The transition $0_2^+ \to 2_2^+$ is allowed by the $(D^\dagger D)^2$ term. However, unlike the SO(6), the strength of this transition is sensitive to such a term. Finally, the common feature for both the SO(7) and SO(6) limits is that $0_3^+ \to 2_1^+$ is strictly forbidden. It is worth noticing that the forbiddeness of the 0_3^+ transitions which deserve some attention here. Naively, since $0_3^+ \to 2_1^+$ is a $\Delta\tau = \pm 1$ transition and the $0_3^+ \to 2_2^+$ is a $\Delta\tau = \pm 2$ transition, the former will be "easier" to break than the latter. In fact this is not true because the breaking of $\Delta\sigma = 0$ selection rule by $(D^\dagger D)^2$ will give the $\Delta\tau = \pm 2$ transition.

Following the above discussion, several identifications can be made for the SO(6) or SO(7) characteristics. First, the branching ratio

$$R = \frac{0_2^+ \to 2_1^+}{0_2^+ \to 2_2^+} \tag{1}$$

offers a good check. If the ratio is very small (say $1 \sim 3\%$ for the typical weak transition), it should reflect that the transition is predominantly SO(6) in nature. If R is large, then it is more SO(7)-like. However, as we have cautioned before, for the SO(7), the strength of the $0_2^+ \to 2_2^+$ transition is dependent on the $\chi(D^\dagger D)^2$ in the E2 operator. In fact we found that the strength of $0_2^+ \to 2_2^+$ increases dramatically with the increase of the χ value (for instance, for $\Omega = 20$ and N=6 case, the ratio is 38% at $\chi = 0.175$). Therefore, a moderate value for this ratio (say $10 \sim 30\%$) can still signal the presence of the SO(7) symmetry or a mixture of SO(6) and SO(7).

Second, the strong transition of $0_3^+ \to 2_2^+$ is predicted in SO(7) symmetry. On the other hand, in the SO(6) limit, the $\Delta\sigma = 0$ selection rule for the 0_3^+ (a $\sigma = N - 2$ state) to 2_1^+ is

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not applicable here because this transition is not allowed in both symmetries. This selection rule for other $\sigma < N$ states (for instance, the excited 2^+ states) should not be considered as prominent evidences to judge whether a nucleus is close to the SO(6) dynamical symmetry or not. Actually, according to the more realistic IBM-2 calculation [15], 2^+ states around 2 MeV are strongly mixed with non-symmetry components. Thus the classification of those 2^+ excited states as $\sigma < N$ cannot be confirmed by more realistic calculation.

In Table 1, we have listed the E2 branching ratios for ^{128}Xe and in Table 2 the absolute B(E2)'s for ^{198}Pt. One sees that the experimentally strong and weak transitions for ^{128}Xe and ^{198}Pt are well accounted for by the SO(7) symmetry. In particular, compare to the other weak transitions, the ratio $R = 14\%$ and 23% for Xe and Pt respectively is an order of magnitude larger. This explicitly shows that at least there is a strong SO(7) component in the wave functions [22]. One may even venture to say that the SO(7) component could be dominant. Note that the B(E2) values of $0_3^+ \to 2_1^+$ in both SO(6) or SO(7) are zero. A Hamiltonain which deviates from SO(6) or SO(7) symmetries will allow this transition to proceed (for instance, see the predictions of the $SO(6)\chi + pairing$ Hamiltonian given by Table 1.)

As shown in Table 1, the predicted strong transition of $0_3^+ \to 2_2^+$ in SO(7) symmetry is consistent with the experimental results for ^{128}Xe (see Table 1). For ^{196}Pt, The FDSM results given in Table 2 also indicate that the transition is not weak. This points to the fact that additional data on the absolute value for this E2 transition is important to confirm the precise nature of SO(7) or SO(6) in ^{196}Pt.

It is worth pointing out that the main reason why Fewell [4] regards that the $U(5)$ symmetry is more appropriate for ^{198}Pt is because SO(6) predicts that all the quadrupole moments must vanish so long that χ=0 in the E2 operator. Within the SO(6), the reproduction of the experimental quadrupole moments would require one to take on a large value of χ, which will spoil the elegance of SO(6) E2 properties (now understood as the SO(5) properties [11]). From Table 2, one sees that both quadrupole moments and B(E2) values are reasonably described under an SO(7)-like Hamiltonian. In Table 2, an IBM-1 calculation

with g-boson is also given. The results imply that the higher states can be further improved by including higher J coherent pair.

To summarize, we have suggested in this paper that the spectroscopes of the nuclei which have hitherto been regarded as strong candidates for the SO(6) dynamical symmetry (i.e. ^{128}Xe and ^{196}Pt) can be better described by the SO(6) plus pairing Hamiltonian. In the FDSM language, this means that these are transitional nuclei between SO(7) and SO(6), rather than good SO(6) nuclei.

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Figure Caption

Fig. 1   The transition of the spectrum from vibrational $(SO(5){\times}SU(2))$ via SO(7) to γ-soft $(SO(6))$. The Hamiltonian is a pairing plus quadrupole one: $H = -0.05(1-\delta)S^{\dagger}S - 0.1\delta P^{2} \cdot P^{2}$ The shell degeneracy Ω is taken to be 20 in the calculation.

Fig. 2   The typical spectroscopes of SO(6) and SO(7) symmetries. The B(E2) values are in units of $B(E2; 2_1^+ \to 0_1^+)$.

TABLE I. Relative B(E2)'s for ^{128}Xe.

transition	exp.[a]	$SO(6)$	$SO(6)_\chi$ + Pairing[b]	SO(7)[c]
$2_2^+ \to 2_1^+$	100	100	100	100
$\to 0_1^+$	1.2	0	1.2	0
$3_1^+ \to 2_2^+$	100	100	100	100
$\to 4_1^+$	37	40	40	40
$\to 2_1^+$	1.0	0	1.3	0
$4_2^+ \to 2_2^+$	100	100	100	100
$\to 4_1^+$	133	91	91	91
$\to 2_1^+$	1.7	0	1.3	0
$0_2^+ \to 2_2^+$	100	100	100	100
$\to 2_1^+$	14	0	1.3	28
$5_1^+ \to 3_1^+$	100	100	100	100
$\to 6_1^+$	204	45	45	45
$\to 4_2^+$	88	46	46	46
$\to 4_1^+$	3.7	0	1.0	0
$0_3^+ \to 2_2^+$	100	0	100	$\neq 0$
$\to 2_1^+$	48	0	40	0

[a]The experimental results are taken from [16,17]

[b]In the calculation is from [11], where the effective charge is e=0.14 (eb), determined from $B(E2; 2_1^+ \to 0_1^+)$ =0.15 (e^2b^2) and χ=0.1.

[c]$\Omega = 20$ and χ=0.2 for SO(7) case.

TABLE II. Comparison of B(E2) values and quadrupole moments for Pt^{196}

$J_i \to J_f$	$B(E2)_{exp}$[a]	$B(E2)_{exp}$[d]	SO(6)	IBM-1 [g]	FDSM[h]
$2_1^+ \to 0_1^+$	0.288(14)	0.274(1)	0.288	0.288	0.286
$4_1^+ \to 2_1^+$	0.403(32)	0.410(6)	0.378	0.393	0.377
$6_1^+ \to 4_1^+$	0.421(116)	0.450(28)	0.384	0.423	0.336
$2_2^+ \to 2_1^+$	0.350(31)	0.370(5)	0.378	0.303	0.394
$2_2^+ \to 0_1^+$	$< 2.0 \times 10^{-6}$ [b]		0	0.004	0.0004
$0_2^+ \to 2_2^+$	0.142(77)	0.1(1)	0.385	0.375	0.261
$0_2^+ \to 2_1^+$	0.033(7) [b]	0.028(5)	0	0.007	0.112
$4_2^+ \to 4_1^+$	0.193(97) [b]	0.084(14)	0.183	0.171	0.191
$4_2^+ \to 2_2^+$	0.177(35) [b]	0.18(2)	0.201	0.199	0.178
$4_2^+ \to 2_1^+$	0.0030(10) [b]	0.001(2)	0	0.004	0.0037
$6_2^+ \to 6_1^+$	0.085(121) [b]		0.108	0.11	0.099
$6_2^+ \to 4_2^+$	0.350(102) [b]		0.232	0.25	0.128
$6_2^+ \to 4_1^+$	0.0037(16) [b]		0	0.001	0.047
$2_3^+ \to 2_1^+$	0.0009(15) [b]		0	7.2×10^{-6}	0.0003
$0_3^+ \to 2_1^+$	< 0.034 [c]		0	0.003	0.0065
$0_3^+ \to 2_1^+$			0		0.216
2_1^+	0.810 ± 0.230[b]	0.62(8)	0	0.671	0.33
4_1^+	$0.389^{+0.302}_{-0.322}$[b]	1.03(12)	0	0.577	0.68
6_1^+	$0.176^{+0.749}_{-0.794}$[b]	−0.18(26)	0	0.412	1.46
2_2^+	$0.303^{+0.258}_{-0.455}$[b]	−0.39(16)	0	0.627	-0.22

[a] $B(E2)_{exp}$ are from [18].

[b] The data are from [19].

[c] The data are [6].

[d] $B(E2)_{exp}$ are from [20].

[g] A modified IBM-1 calculation with g-boson [21]

[h]The FDSM calculation is carried out according to [10], where a pairing+QQ Hamiltonian for a proton-neutron system is employed, and $e_\pi = 0.195(eb)$ and $e_\nu = 0.183(eb)$ for E2 operator.

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